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Dispersed phase of particles in rotating turbulent fluid flows

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Certain effects, caused by rotating turbulent fluid flows in the presence of gravitational force, for transport of particles dispersed in fluid are suggested and quantified through kinetic or probability density function approach based macroscopic equations. These results are *exact* when turbulent fluctuations in fluid velocity along the particle path have Gaussian distribution.

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The frequent existence of two-phase particle/droplet laden turbulent flows in nature and manmade systems continues to intrigue both physicists and engineers in developing predictive theory/model for their complete description. A variety of phenomena related to dispersed phase of particles in turbulent flows have been discovered, such as, turbophoresis [1], preferential distribution [2], turbulent thermal diffusion, barodiffusion, anomaly, and intermittency [3]. A recent study by Elperin *et al.* [4] further suggests effects of fast rotation of fluid on the dynamics of dispersed phase suspended in turbulent fluid flows which has relevance in important industrial and natural situations, such as, protoplanetary nebula and disks [5]. Despite many advancements [6], it remains challenging to come up with a unique predictive theory/model useful for engineering purposes and also capable in quantifying various intrinsic effects and phenomena. In recent years, the probability density function (PDF) approach has shown promise for quantitative predictions and in capturing interesting phenomena [7–11]. In continuation of our previous efforts on the PDF approach [9–12] in unifying different aspects of particle/droplet-laden turbulent flows, this letter considers application of the PDF for accurately quantifying effects in the situation of particle laden rotating turbulent flows.

We consider particles dispersed in rotating fluid flows moving under the action of fluid drag force, forces due to rotation (Coriolis and centrifugal), and gravity. The trajectory of each individual particle is governed by the Lagrangian equations for particle position X_i (also denoted by $\mathbf{X}$) and velocity V_i (also denoted by $\mathbf{V}$):

$$\frac{dX_i}{dt} = V_i, \quad \frac{dV_i}{dt} = \beta_v(U_i - V_i) + 2\epsilon_{iab}V_a\Omega_b + f_i, \tag{1}$$

described in a coordinate system rotating with constant angular velocity Ω_b and the subscript $i=1,2,3$ represents the components of the vector. Here, β_v is the inverse of the particle velocity time constant τ_p defined based on the Stokes drag, U_i is the carrier fluid velocity in the vicinity of the particle, ϵ_{iab} is the Levi-Civita's alternating tensor, and $f_i = f_i^c + f_i^g$ accounts for centrifugal f_i^c and gravitational f_i^g accelerations. Here f_i^c and f_i^g are components of $\mathbf{f}^c = -\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$ and $\mathbf{f}^g = f^g\mathbf{r}$, respectively, with $f^g = -(GM/r^3)$ and position vector of particle $\mathbf{r}$ and $r=|\mathbf{r}|$ for the situation of protoplanetary nebula or disks. Also, G is the gravitational constant and M is the mass responsible for gravitational force. Extracting the information for collective behavior of particles from the Lagrangian Eq. (1) requires statistical approaches, such as, Reynolds averaged Navier-Stokes (RANS) type and PDF approaches.

The RANS approach aims at obtaining the closed average, either (1) nonweighted average or (2) density-weighted average [13], equations from the instantaneous equations for number density $n(\mathbf{x},t)$ and velocity field $V_i(\mathbf{x},t)$ of the dispersed phase in physical space $\mathbf{x}$ and time t, e.g., see Ref. [14]. For the situation governed by Eq. (1), instantaneous equations are

$$\partial n/\partial t + \partial(nV_i)/\partial x_i = 0, \tag{2}$$

$$\frac{\partial V_i}{\partial t} + V_j\frac{\partial V_i}{\partial x_j} = \beta_v(U_i - V_i) + 2\epsilon_{iab}V_a\Omega_b + f_i, \tag{3}$$

and the nonweighted averaging of these are written as

$$\partial N/\partial t + \partial(N\langle V_i\rangle)/\partial x_i + \partial\langle nv_i''\rangle/\partial x_i = 0, \tag{4}$$

$$\frac{\partial\langle V_i\rangle}{\partial t} + \langle V_j\rangle\frac{\partial\langle V_i\rangle}{\partial x_j} + \left\langle v_j''\frac{\partial v_i''}{\partial x_j}\right\rangle = \beta_v(\langle U_i\rangle - \langle V_i\rangle) + 2\epsilon_{iab}\langle V_a\rangle\Omega_b + f_i. \tag{5}$$

Here $\langle\ \rangle$ denotes ensemble average, v_i'' is fluctuating part of $V_i = \langle V_i\rangle + v_i''$, $N=\langle n\rangle$ and $\langle V_i\rangle$ represent mean density and mean velocity, respectively. It should be noted that molecular diffusion and internal stresses of particle are neglected while writing Eqs. (2)–(5). The internal stresses become important for finite size particles and their modeling have been under persistent investigation by researchers (e.g., see Ref. [15], and references cited therein). In this paper, these stresses are not included as they do not arise in the PDF approach where particles are assumed as point particles.

Now, the unknown terms $\langle nv_i''\rangle$ and $\langle v_j''\partial v_i''/\partial x_j\rangle$ pose closure problems in Eqs. (4) and (5), respectively. By using stochastic calculus for the Wiener process, Elperin *et al.* [16,17] derived the expression for $\langle nv_i''\rangle$ under the presence

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of molecular diffusion and when v_i'' is correlated over a very short period of time. The expression is

$$\langle n v_i'' \rangle = -\tau N \langle v_i'' \partial v_j'' / \partial x_j \rangle - \tau \langle v_i'' v_m'' \rangle \partial N / \partial x_m, \qquad (6)$$

where τ is the momentum relaxation time of random velocity field v_i'' and the term $\langle v_i'' \partial v_j'' / \partial x_j \rangle$ is shown to be responsible for two phenomena of turbulent thermal diffusion and barodiffusion in compressible fluid flow [17] and effects in fast rotating turbulent flows [4]. Recently, it is shown that the two phenomena associated with $\langle v_i'' \partial v_j'' / \partial x_j \rangle$ disappear for $\tau = 1/\beta_v$ and this disappearance is absent in the PDF approach based density-weighted average equations for velocity $\overline{V_i}$ [9]. Extending the argument given in Ref. [9] suggests that the effects in rotating turbulence obtained by Elperin *et al.* [4] in the framework of nonweighted average would also vanish for $\tau = 1/\beta_v$ and which seems to be dubious. And we think the density-weighted average based analysis more appropriate.

PDF approach: The density-weighted average (denoted by overbar) of Eqs. (2) and (3) yields

$$\partial N / \partial t + \partial (N \overline{V_j}) / \partial x_j = 0, \qquad (7)$$

$$\beta_v \overline{V_j} + \frac{\partial \overline{V_j}}{\partial t} + \overline{V_i} \frac{\partial \overline{V_j}}{\partial x_i} + \frac{\partial \overline{v_n' v_j'}}{\partial x_n} = \beta_v \langle U_j \rangle + \{ 2 \epsilon_{jab} \overline{V_a} \Omega_b + f_j \} - [\overline{v_n' v_j'}] \frac{\partial}{\partial x_n} \ln N + \beta_v \overline{u_j''}, \qquad (8)$$

where for any instantaneous variable A, $\overline{A} = \langle nA \rangle / N$, a' is fluctuation in $A = \overline{A} + a' = \langle A \rangle + a''$ over $\overline{A}$. Also $\langle a' \rangle \neq 0$ and $\overline{a''} \neq 0$. The unknown terms $\overline{v_n' v_j'}$ and, in particular, $\overline{u_j''}$ are difficult to model from the instantaneous equations. Whereas, the expression for $\overline{u_j''}$ and equations for other unknown density-weighted terms can be obtained with ease in PDF approach [7,8,10].

Considering particles as point particles, use of Liouville theorem and Lagrangian Eq. (1) allow us to write the governing equation for ensemble average of the phase space density $W(\mathbf{x}, \mathbf{v}, t)$ of the particles, written as

$$\left\{ \frac{\partial}{\partial t} + \frac{\partial}{\partial x_i} v_i + \frac{\partial}{\partial v_i} w_i \right\} \langle W \rangle = - \frac{\partial}{\partial v_i} [\beta_v \langle u_i'' W \rangle], \qquad (9)$$

with $w_i = \beta_v \langle U_i \rangle - \beta_v v_i + 2 \epsilon_{iab} v_a \Omega_b + f_i$. Here $\mathbf{x}$ and $\mathbf{v}$ are phase space variables corresponding to $\mathbf{X}$ and $\mathbf{V}$, respectively, and u_i'' represents fluctuations in fluid velocity in the vicinity of particle over the mean velocity $\langle U_i \rangle$. In the kinetic or PDF Eq. (9), the unknown term $\langle u_i'' W \rangle$ poses a closure problem which can be tackled by various methods available [7,8,10]. Here by employing the Furutsu-Novikov-Donsker functional formula [18,12], the expression for $\langle u_i'' W \rangle$ can be obtained as

$$\beta_v \langle u_i'' W \rangle = - [\partial \lambda_{ki} / \partial x_k + \partial \mu_{ki} / \partial v_k - \gamma_i] \langle W \rangle, \qquad (10)$$

where various tensors are

$$\lambda_{ki} = \beta_v^2 \int_0^t dt_2 \langle u_i'' u_j''(t|t_2) \rangle G_{jk}(t_2|t),$$

$$\gamma_i = \beta_v^2 \int_0^t dt_2 \left\langle \frac{\partial u_i''}{\partial x_k} u_j''(t|t_2) \right\rangle G_{jk}(t_2|t), \qquad (11)$$

$$\mu_{ki} = \beta_v^2 \int_0^t dt_2 \langle u_i'' u_j''(t|t_2) \rangle \frac{d}{dt} G_{jk}(t_2|t), \qquad (12)$$

and shorthand notation $u_j''(t|t_2)$ is used to represent $u_j''(\mathbf{x}, \mathbf{v}, t|t_2)$ which is the fluid velocity fluctuation at time t_2 in the vicinity of the particle that passes through $\mathbf{x}$ with velocity $\mathbf{v}$ at time t. Also u_i'' represents $u_i''(\mathbf{x}, t)$ in Eqs. (11) and (12) and now onwards. The equation for $G_{ij}(t_2|t)$, $\forall t \geq t_2$, in the presence of rotation and f_i becomes

$$\frac{d^2}{dt^2} G_{jk}(t_2|t) + \beta_v \frac{d}{dt} G_{jk} - \beta_v G_{ji} \frac{\partial \langle U_k \rangle}{\partial x_i} - 2 \epsilon_{kab} \Omega_b \frac{d}{dt} G_{ja} - \frac{\partial f_k}{\partial x_i} G_{ji} = \delta_{jk} \delta(t - t_2). \qquad (13)$$

For later convenience, expressions for λ_{ij} and γ_j are now written in different forms

$$\frac{\lambda_{ij}}{\beta_v} = \int_0^t ds \langle u_j'' \Delta v_i \rangle, \quad \frac{\gamma_j}{\beta_v} = \int_0^t ds \left\langle \frac{\partial u_j''}{\partial x_i} \Delta v_i \right\rangle, \qquad (14)$$

where $\Delta v_i = \int_0^s dt_2 \beta_v u_k''(t|t_2) [dG_{ki}(t_2|s)/ds]$ represents change in particle velocity due to $\beta_v u_k''(t|t_2)$, during time 0 to s, along the trajectory that passes through $\mathbf{x}, \mathbf{v}$ at time t. Also $u_j'' \equiv u_j''(\mathbf{x}, t)$ in Eq. (14).

The Eqs. (10)–(13) are *exact* closure solution for particle phase when u_i'', along the particle path, has Gaussian distribution and its statistical properties $\langle u_i'' u_j''(t|t_2) \rangle \equiv u_{ij}$ and $\langle \partial u_i'' / \partial x_k u_j''(t|t_2) \rangle$ are known. For non-Gaussian behavior, third order correlations of u_i'' would appear in the closure solution and which are taken zero in the present analysis. The equation for u_i'':

$$du_i''/dt = -\rho^{-1} \partial p'' / \partial x_i + \nu \nabla^2 u_i'' + u_a'' R_{ia} + R_i, \qquad (15)$$

where ρ is fluid density, p'' is fluctuation in fluid pressure, ν is kinematic viscosity of fluid, $R_{ia} = [(\partial \langle U_i \rangle / \partial x_a) + 2 \epsilon_{iab} \Omega_b]$ and $R_i = (V_j - U_j)(\partial u_i'' / \partial x_j) + \langle u_j'' (\partial u_i'' / \partial x_j) \rangle$. To obtain closed equation for u_{ij}, we substitute in Eq. (15) model $-(1/\rho_f) \times (\partial p'' / \partial x_i) + \nu \nabla^2 u_i'' = -(u_i'' / \widetilde{T}_L) + W_i$ where $\widetilde{T}_L$ is integral time scale, W_i is a white noise term (e.g., see Ref. [19]), neglect $(V_j - U_j)(\partial u_i'' / \partial x_j)$ which is strictly valid for particles with small time constant so $V_i \approx U_i$, multiply Eq. (15) by $u_j''(t|t_2)$ and take ensemble average. The resulting equation is

$$\frac{du_{ij}}{dt} = -\frac{u_{ij}}{\widetilde{T}_L} + \langle u_a'' u_j''(t|t_2) \rangle \left(\frac{\partial \langle U_i \rangle}{\partial x_a} + 2 \epsilon_{iab} \Omega_b \right), \qquad (16)$$

from which we obtain

$$\langle u_i'' u_j''(t|t_2)\rangle = \langle u_i''(t|t_2) u_j''(t|t_2)\rangle e^{-(t-t_2)/\tilde{T}_L} + u_{ij}^1, \qquad (17)$$

with $u_{ij}^1 = \langle u_a''(t|t_2) u_j''(t|t_2)\rangle [2\tilde{T}_L \epsilon_{iab}\Omega_b\{1-e^{-[(t-t_2)/\tilde{T}_L]}\} + \int_{t_2}^t ds e^{[(s-t)/\tilde{T}_L]}\partial\langle U_i\rangle/\partial x_a]$, by using perturbation expansion around the first term on the right-hand side (rhs) of Eq. (16). Later for discussion, we would consider only the first exponential term in Eq. (17).

Using Eqs. (9) and (10), the macroscopic equation for $N = \int\langle W\rangle d\mathbf{v}$ is obtained identical to Eq. (7). The equation for density-weighted average of particle velocity $\overline{V_j} = (1/N)\int v_j\langle W\rangle d\mathbf{v}$ is identical to Eq. (8) but with closure solution for $\beta_v \overline{u_j''}$ written as

$$\beta_v \overline{u_j''} = -\overline{\lambda_{ij}}\frac{\partial}{\partial x_i}\ln N - \frac{\partial}{\partial x_i}\overline{\lambda_{ij}} + \overline{\gamma_j} \qquad (18)$$

and which accounts for interactions between fluid turbulence structures and particles. The macroscopic equation for $\overline{v_j' v_n'} = (1/N)\int(v_j - \overline{V_j})(v_n - \overline{V_n})\langle W\rangle d\mathbf{v}$, in Eq. (8), is

$$\frac{\partial\overline{v_j' v_n'}}{\partial t} + \overline{V_i}\frac{\partial}{\partial x_i}\overline{v_j' v_n'} + \frac{1}{N}\frac{\partial}{\partial x_i}[N\overline{v_i' v_j' v_n'}] = -\overline{v_i' v_j'}\frac{\partial\overline{V_n}}{\partial x_i} - \overline{v_i' v_n'}\frac{\partial\overline{V_j}}{\partial x_i}$$
$$- 2\beta_v\overline{v_j' v_n'} - \overline{\lambda_{kj}}\frac{\partial\overline{V_n}}{\partial x_k} - \overline{\lambda_{kn}}\frac{\partial\overline{V_j}}{\partial x_k} + \overline{\mu_{jn}} + \overline{\mu_{nj}}$$
$$+ 2\Omega_b[\epsilon_{jab}\overline{v_a' v_n'} + \epsilon_{nab}\overline{v_a' v_j'}]. \qquad (19)$$

While writing Eqs. (18) and (19), the density weighted average of various tensors is considered to be equal to their respective instantaneous values, e.g., $\overline{\lambda_{ij}} = \lambda_{ij}$.

Various terms in Eqs. (8), (18), and (19), suggest different phenomena related to the transport of the particle phase. The term $\beta_v^{-1}\partial\overline{v_j' v_n'}/\partial x_i$ in Eq. (8) accounts for the turbophoresis [1,7] phenomenon. The effects of rotation and gravity on the turbophoresis can be quantified through Eq. (19). The last term on the rhs of Eq. (19) exhibits explicitly the effect of rotation caused by the Coriolis force. Further, rotation and gravity affect $\overline{v_j' v_n'}$ through $\overline{\lambda_{ij}}$ and $\overline{\mu_{ij}}$ which are functionals of G_{ij}.

The term $2\beta_v^{-1}\epsilon_{jab}\overline{V_a}\Omega_b$ in Eq. (8), which is due to the mean Coriolis force, contributes to the drift velocity of the particle phase and f_j/β_v represents the drift velocity due to centrifugal and gravitational forces. The first term on the rhs of Eq. (18) is responsible for turbulent diffusion of particles. The last two terms in Eq. (18) contains interesting phenomena for drift velocity of the particle when fluid variables u_i'' along the particle path are correlated with finite time. Because for correlations having delta function in time, these two terms reduce to zero as $G_{jk}(t|t) = 0$ [9]. For finite correlation time, we describe the phenomena for cases of (1) slow rotation, i.e., Ω_b is small and (2) fast rotation.

For slow rotation, using the first term in Eq. (17) for $\langle u_i'' u_j''(t|t_1)\rangle$, expanding the solution of Eq. (13) as $G_{jk} = G_{jk}^0 + G_{jk}^1$ and using expressions for tensors given by Eqs. (11) with incorporating approximation $\langle u_i''(t|t_1) u_j''(t|t_1)\rangle \cong \langle u_i'' u_j''\rangle$, we obtain

$$-\frac{\partial\lambda_{kj}}{\partial x_k} + \gamma_j \cong -\beta_v^2\int_0^t dt_1\langle u_j'' u_k''(t|t_1)\rangle\frac{\partial G_{ki}(t_1|t)}{\partial x_i}$$
$$-\beta_v^2\int_0^t dt_1 G_{ki}^1(t_1|t)e^{-(t-t_1)/\tilde{T}_L}\left\langle\frac{\partial u_k''}{\partial x_i}u_j''\right\rangle$$
$$-\left\langle\frac{\partial u_k''}{\partial x_k}u_j''\right\rangle\beta_v\left\{\tilde{T}_L[1-e^{(-t/\tilde{T}_L)}]\right.$$
$$\left. + \frac{\tilde{T}_L}{\beta_v\tilde{T}_L + 1}[e^{-t(\beta_v + 1/\tilde{T}_L)} - 1]\right\}. \qquad (20)$$

Here $G_{jk}^0 = G^0(t_2|t)\delta_{jk} = \delta_{jk}[1 - e^{-\beta_v(t-t_2)}]/\beta_v$ and

$$\frac{d^2}{dt^2}G_{jk}^1(t_2|t) + \beta_v\frac{d}{dt}G_{jk}^1 - \beta_v G_{ji}^0\frac{\partial\langle U_k\rangle}{\partial x_i} - 2\epsilon_{kab}\Omega_b\frac{d}{dt}G_{ja}^0$$
$$-\frac{\partial f_k}{\partial x_i}G_{ji}^0 = 0, \qquad (21)$$

and u_j'', u_k'' represent $u_j''(\mathbf{x},t)$, $u_k''(\mathbf{x},t)$, respectively. The last term on the rhs of Eq. (20) gives the phenomena of turbulent thermal diffusion and barodiffusion [9] for compressible flows. Substituting solution for G_{ki}^1 from Eq. (21) into Eq. (20), the second term on the rhs becomes equal to

$$\beta_v^2\left\langle\frac{\partial u_k''}{\partial x_i}u_j''\right\rangle\left[-\beta_v\frac{\partial\langle U_i\rangle}{\partial x_k}A_1 - 2\epsilon_{ikb}\Omega_b A_2 - \frac{\partial f_i}{\partial x_k}A_1\right], \qquad (22)$$

where $A_1 = \int_0^t dt_1[e^{-[(t-t_1)/\tilde{T}_L]}\int_{t_1}^t dt_2 G^0(t_2|t)G^0(t_1|t_2)]$ and $A_2 = \int_0^t dt_1[e^{-[(t-t_1)/\tilde{T}_L]}\int_{t1}^t dt_2 G^0(t_2|t)(d/dt_2)G^0(t_1|t_2)]$. The first term in Eq. (22) represents the contribution to particle phase velocity due to the shear rate. The second term in Eq. (22) can also be written in the form $2\beta_v^2\langle\Omega_b\epsilon_{bik}[\partial U_k(\mathbf{x},t)/\partial x_i]u_j''\rangle A_2$. This form suggests that the component of fluid vorticity at particle location $\{\epsilon_{bik}[\partial U_k(\mathbf{x},t)/\partial x_i]\}$ in the direction of rotation Ω_b produces a drift velocity for particle phase and whose origin is the Coriolis force. This phenomenon was suggested by Elperin *et al.* [4] in the limiting case of fast rotation and when the fluctuations in particle phase velocity is correlated over a short period in time. It should be noted that the present derivation has no such limitations for particle velocity.

The last term in Eq. (22) represents the interactions of centrifugal and gravitational forces acting on the particle with turbulence velocity fluctuations at the particle location. For the discussion purpose, we consider a finite value for rotation only about x_3 axis, i.e., $\Omega_1 = \Omega_2 = 0$ and $\Omega_3 \neq 0$. Then, the centrifugal part of f_k is given as $f_k^c = x_k\Omega_3^2 - \delta_{k3}x_k\Omega_3^2$, and its contribution to the last term is equal to

$$-\beta_v^2 A_1\Omega_3^2[\langle u_j''\partial u_k''/\partial x_k\rangle - \langle u_j''\partial u_3''/\partial x_3\rangle]. \qquad (23)$$

In case of a compressible flow at low Mach number of an ideal gas obeying the equation of state $P = \rho RT$:

$$\langle u_i''\partial u_j''/\partial x_j\rangle \cong \langle u_i'' u_j''\rangle\partial[\ln\langle T\rangle - \ln\langle P\rangle]/\partial x_j, \qquad (24)$$

where P, T, and ρ are the pressure, temperature, and density of the gas, respectively, and R is the gas constant [9,10].

Thus, the first term in square brackets in Eq. (23) contributes to the phenomena of turbulent thermal diffusion and barodiffusion in compressible flows and the second term is nonzero for incompressible turbulent flow. The contribution of the gravitational part $f_k^g = f^g x_k$ of f_k to the last term in Eq. (22) is

$$-\beta_v^2 f^g \langle u_j'' \partial u_k'' / \partial x_k \rangle A_1 - \beta_v^2 x_i \frac{\partial f^g}{\partial x_k} \langle u_j'' \partial u_k'' / \partial x_i \rangle A_1. \qquad (25)$$

Here the first term vanishes and the second term is nonzero in incompressible flows. Equation (24) suggests that the first term in Eq. (25) contributes to the phenomena of turbulent thermal diffusion and barodiffusion of the dispersed phase in compressible flow of an ideal gas. The second term in Eq. (25) is nonzero due to the finite variation of f^g in space.

Now we show for fast rotation and without gravitational effect, drift velocity of particle exhibits trend similar to that suggested by Elperin *et al.* [4]. For large value of time t, contributions to λ_{ij} and γ_j in Eq. (14) come from the correlations of $u_j''(\mathbf{x},t)$ and $\partial u_j''(\mathbf{x},t)/\partial x_i$ with velocity of particles, between $s(<t)$ and t, present in the region near to $\mathbf{x}$ and t and passing through $\mathbf{x}$ at time t. Assuming exponential form for these correlations with integral time scale $\tilde{T}_L$ [consistent with Eq. (15)], the last two terms in Eq. (18) simplify to

$$\beta_v^{-1}\left[-\frac{\partial \lambda_{kj}}{\partial x_k} + \gamma_j\right] \approx -\left\langle u_j'' \frac{\partial V_i(\mathbf{x},t)}{\partial x_i}\right\rangle \int_0^t ds e^{(s-t)/\tilde{T}_L}, \qquad (26)$$

and represent drift velocity. Under the fast rotation without gravity, Elperin *et al.* [4] obtained an approximate relation $[\partial V_i(\mathbf{x},t)/\partial x_i] \approx [2\tau_p \Omega_i \epsilon_{iab}(\partial U_b/\partial x_a)/(1+4\tau_p^2 \Omega_k \Omega_k)]$. Using it in Eq. (26) then provides the dependence of the drift velocity on $\mathbf{\Omega}$ similar to that obtained by Elperin *et al.* It should be noted that this drift velocity does not vanish when $\tau = 1/\beta_v$ which is the case in the framework of nonweighted average, used by Elperin *et al.* [4], as explained earlier in this paper.

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